



Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at <http://about.jstor.org/participate-jstor/individuals/early-journal-content>.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

The limits of y are 0 and $\frac{p}{2+\sqrt{2}}=y'$.

$$\begin{aligned} \therefore \Delta &= \frac{\frac{1}{2}\pi \int_0^{y'} \frac{y^2(p-2y)^2}{(p-y)^2} dy}{\int_0^{y'} dy} = \frac{\pi(2+\sqrt{2})}{4p} \int_0^{y'} \left(4y^2 + 4py + 5p^2 + \frac{p^4}{(p-y)^2} - \frac{6p^3}{p-y} \right) dy \\ &= \frac{\pi p^2}{12} [27 - 4\sqrt{2} - 9(2+\sqrt{2})\log 2]. \end{aligned}$$

In this solution, as in solution of problem 75, I used the limits of y , 0 and $p/(2+\sqrt{2})$. These limits give all possible variations of size of area. Any other areas are mere repetitions of those included in the above and such a repetition or doubling of areas I believe to be inadmissible.

94. Proposed by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

Three points are taken at random on the surface of the sphere. Find the chance that the triangle thus formed is acute angled.

Solution by the PROPOSER.

Let AD be the diameter of the section of the sphere made by the plane through the three random points A, B, C ; M its center; O the center of the sphere; OP a line such that AB is parallel to the plane MOP ; p =the chance.

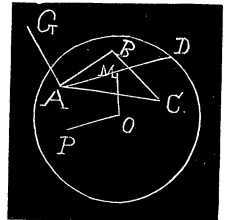
Let $AO=r$, $\angle AOM=\theta$, $\angle GAC=\varphi$, $\angle GAB=\psi$, $\angle MOP=\lambda$, the angle MOP makes with some fixed plane through $OP=\rho$.

An element of the sphere at A is $4\pi r^2 \sin\theta d\theta$; at B , $4r^2 \sin\theta \sin(\varphi-\psi) \sin\psi \sin\lambda d\psi d\rho$; at C , $4r^2 \sin\theta \sin\varphi d\varphi d\lambda$.

The limits of θ are 0 and $\frac{1}{2}\pi$; of φ , $\frac{1}{2}\pi$ and π ; of ψ , $\pi-\varphi$ and $\frac{1}{2}\pi$; of λ , 0 and π ; of ρ , 0 and 2π .

The three points can be taken $64\pi^3 r^6$ ways on the surface of the sphere. Hence

$$\begin{aligned} p &= \frac{1}{64\pi^3 r^6} \int_0^{\frac{1}{2}\pi} \int_{\frac{1}{2}\pi}^{\pi} \int_{\pi-\varphi}^{2\pi} \int_0^{\pi} \int_0^{2\pi} 4\pi r^2 \sin\theta d\theta \cdot 4r^2 \sin\theta \sin\varphi d\varphi d\lambda \\ &\quad \times 4r^2 \sin\theta \sin(\varphi-\psi) \sin\psi \sin\lambda d\psi d\rho \\ &= \frac{2}{\pi} \int_0^{\frac{1}{2}\pi} \int_{\frac{1}{2}\pi}^{\pi} \int_{\pi-\varphi}^{\pi} \int_0^{\pi} \sin^3 \theta \sin\varphi \sin\psi \sin(\varphi-\psi) \sin\lambda d\theta d\varphi d\psi d\lambda \\ &= \frac{4}{\pi} \int_0^{\frac{1}{2}\pi} \int_{\frac{1}{2}\pi}^{\pi} \int_{\pi-\varphi}^{\pi} \sin^3 \theta \sin\varphi \sin\psi \sin(\varphi-\psi) d\theta d\varphi d\psi \end{aligned}$$



$$\begin{aligned}
&= \frac{1}{\pi} \int_0^{\frac{1}{2}\pi} \int_{\frac{1}{2}\pi}^{\pi} \sin^2 \theta (4 \sin^2 \varphi - 4 \sin^4 \varphi + \pi \sin \varphi \cos \varphi - 2 \varphi \sin \varphi \cos \varphi) d\varphi d\theta \\
&= \frac{1}{2} \int_0^{\frac{1}{2}\pi} \sin^3 \theta d\theta = \frac{1}{2}.
\end{aligned}$$

MISCELLANEOUS.

88. Proposed by COOPER D. SCHMITT, A. M., Professor of Mathematics, University of Tennessee, Knoxville, Tenn.

$$\text{Solve to infinity the series } 5\cos\theta = \frac{7\cos 3\theta}{3!} + \frac{9\cos 5\theta}{5!} + \dots$$

Solution by the PROPOSER.

$$\text{Let the series be } C = \frac{5\cos\theta}{1} + \frac{7\cos 3\theta}{3!} + \frac{9\cos 5\theta}{5!} + \dots$$

$$\text{also, } S = \frac{5\sin\theta}{1} + \frac{7\sin 3\theta}{3!} + \frac{9\sin 5\theta}{5!} + \dots$$

$$\text{Then } C + Si = \frac{5(\cos\theta + i\sin\theta)}{1} + \frac{7(\cos 3\theta + i\sin 3\theta)}{3!} + \dots$$

or using a familiar notation,

$$C + Si = \frac{5e^{i\theta}}{1} + \frac{7e^{3i\theta}}{3!} + \frac{9e^{5i\theta}}{5!} + \dots$$

which can be written thus :

$$\begin{aligned}
&= 4\left(e^{i\theta} + \frac{e^{3i\theta}}{3!} + \frac{e^{5i\theta}}{5!} + \dots\right) + \left(e^{i\theta} + \frac{3e^{3i\theta}}{3!} + \frac{5e^{5i\theta}}{5!} + \dots\right) \\
&= 4\left(e^{i\theta} + \frac{e^{3i\theta}}{3!} + \frac{e^{5i\theta}}{5!} + \dots\right) + e^{i\theta}\left(1 + \frac{e^{2i\theta}}{2!} + \frac{e^{4i\theta}}{4!} + \dots\right) \\
&= 2\left(e^{e^{i\theta}} - e^{-e^{i\theta}}\right) + \frac{e^{i\theta}}{2}\left(e^{e^{i\theta}} + e^{-e^{i\theta}}\right)
\end{aligned}$$

But $e^{i\theta} = \cos\theta + i\sin\theta$, so that we have

$$\begin{aligned}
C + Si &= 2\left(e^{\cos\theta + i\sin\theta} - e^{-\cos\theta - i\sin\theta}\right) + \frac{e^{i\theta}}{2}\left(e^{\cos\theta + i\sin\theta} + e^{-\cos\theta - i\sin\theta}\right) \\
&= 2e^{\cos\theta} e^{i\sin\theta} - 2e^{-\cos\theta} e^{-i\sin\theta} + \frac{e^{\cos\theta}}{2} \cdot e^{i\theta + i\sin\theta} + \frac{e^{-\cos\theta}}{2} e^{i\theta - i\sin\theta}
\end{aligned}$$